Optimized Blood Pressure Control during Surgery

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***Abstract*:** The present work describes the design and analysis of a blood pressure control system during surgery using anesthesia in an efficient way. The total system under consideration is supposed to consist of the patient's body dynamics, suitable compensator, and surgical disturbance operated in the closed loop manner with unity negative feedback yielding controlled blood pressure as output. With appropriate control of valve setting of the anesthetic reagent chamber, the vapor of the reagent is emanated. This vapor emanation as the process is concerned is further mingled with surgical disturbance where the output is fed to the patient for controlled dose of anesthesia. With the application of controlled dose of anesthesia, the regulated blood pressure of the patient becomes achievable. As the output which is fed back in the unity negative feedback path to the compensator input of an error detector whose other input receives the desired blood pressure for the study of the patient is now pressure track under desired environment. The optimality of the performance for the system is considered to be attained with proportional gain of one proportional plus integral (PI) compensator, so chosen that the integral square error becomes a minimum. The present work considers the minimization using particle swarm optimization (PSO) which searches the maxima of the reciprocal of the ISE with in the range of the value of the system gain as obtained by Routh Hurwitz criterion. The overall system is found to be stable, controllable and observable. The system is also analyzed in sampled data control domain (z domain). The stability in z domain is analyzed using Jury's Stability test. The overall system offers an in-depth sight for the biomedical engineering application of a blood pressure control system operated in optimal condition. MATLAB software is appropriately used in the entire analysis.

***Keywords*:** Stability, PI compensator, Integral Square Error, Sample data control

1. Introduction

Control of blood pressure during surgery using anesthesia is extremely important matter in medical science, because the blood pressure of a patient during surgery using anesthesia normally varies over a range, although the safe surgery demands the maintenance of constant mean arterial pressure to some requisite value. The level of arterial blood pressure is postulated to be a proxy for depth of anesthesia during surgery. The level of anesthesia is required to be controlled for healthy environment of the surgery for controlled level of mean arterial pressure

A block diagram of the system is shown in Figure: 1[1, 2], where the impact of surgery is represented by the disturbance D(s).

In the system one compensator which controls the overall system for appropriate performance criteria of the system is used.
The compensator considers the adaptive noise cancellation method towards the system design for having a desired control system.

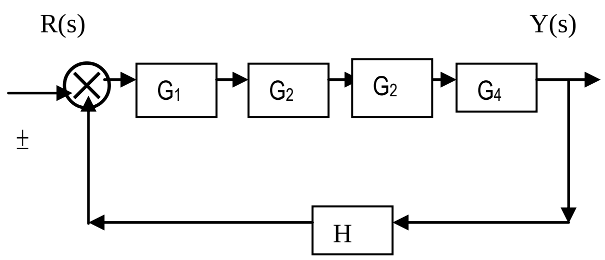

Figure 1: Block Diagram of blood pressure control system.

$G_1(s) = (ks+k_1) / s$; $G_2(s)=1/s$ $G_3(s) =1/s$; $G_4(s) = 1/(s+2)^2$;
$H(s) =1$;
R(s) =Desired blood pressure, Y(s) =Actual blood pressure.

2. Blood pressure control

There is wide range of blood pressure in healthy subjects. Increase in blood pressure occurs with age. In the in the same individual, transient variations in blood pressure is common [3], nervousness, excitement, exertion, fatigue, cold and fatigue may raise the normal level to some extent but in these conditions, systolic pressure is affected more than the diastolic one.

The systolic blood pressure is mainly determined by the force of contraction of the left ventricle. The diastolic blood pressure is regulated by the arteriolar resistance, which converts the intermittent output of the heart into a continuous capillary blood flow. During systole the large musculo-elastic arteries are distended and during diastole their elastic recoil helps to maintain the arterial pressure.

The arterial pressure consists of systolic and diastolic pressures. The mean arterial pressure is the average arterial pressure throughout each cardiac cycle of the heart beat. The arterial pressure usually remains nearer to diastolic level than to systolic level during a greater portion of the pulse cycle.

The mean arterial pressure is lowest immediately after the birth, measuring about 70 mm of Hg at birth and reaching an average of amount 110 mm Hg in the normal old person and as high as 130 mm Hg in person with arteriosclerosis [4].During surgery due to loss of blood, the mean arterial pressure falls (Figure 2:).The blood pressure is, however, very importantly required to be maintained to specific safe value for normal surgery operation., The surgery operation using anesthesia again having other contribution to changing blood pressure. So the anesthesia dose is ultimately required to be controlled for safe surgical environment.

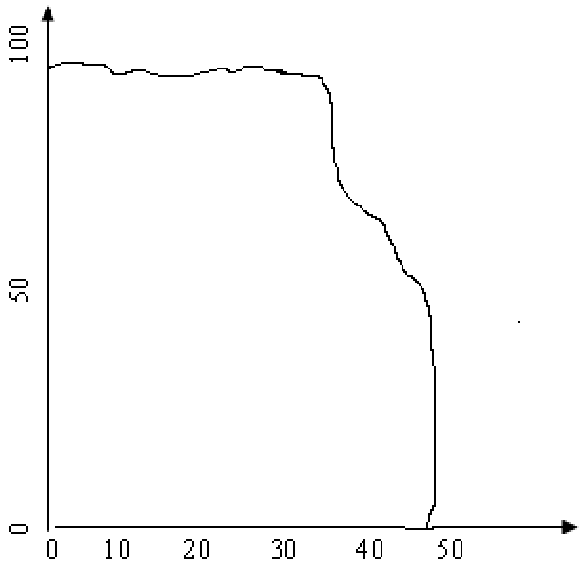

Figure 2: Hemorrhage effect on arterial pressure.

3.Conversion from s-domain to z-domain [5]:

Z-transform helps in the analysis and design of sample data control system, as Laplace transform does in the analysis and design of continuous data control system.

The z-transform $F(z)$ of a sample data control signal $f(KT)$ is defined by the relation:

$$F(z) = \sum_{K=0}^{\infty} f(KT) z^{-K} \, . \qquad \dots (1)$$

The above relation is derived from the Laplace transformation as applied to sample data control signal.

Assuming $e^{sT} = z$ be the concerned transformation variable in Laplace Transformation,

we have

$$sT = \ell n z$$

i.e. $$s^{-1} = \frac{T}{\ell n z} \qquad \dots (2)$$

Using power series expansion of $\ell n z$, the above equation becomes:

$$s^{-1} = \frac{T}{2}\left[\frac{1}{u} - \frac{4}{3}u - \frac{4}{45}u^3 - \frac{44}{945}u^5 \dots\dots\dots\right] \qquad \dots (3)$$

Where

$$u = \frac{1-z^{-1}}{1+z^{-1}} \qquad \dots (4)$$

In general, for any positive integral value of n

$$s^n = \left(\frac{T}{2}\right)^n \left[\frac{1}{u} - \frac{1}{3}u - \frac{4}{45}u^3 - \frac{44}{945}u^s\right]^n \qquad \dots (5)$$

By using binomial expansion in the above equation for various values of n, we may have the transformation from s to z domain.

4. Integral square error (J) [6]:

Instead of the time domain calculation of J (integral square error), the complex frequency domain can be used. According to a theorem in mathematics by Parseval

$$J = I_{SE} = \int_0^{\infty} e^2(t)dt = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} E(s)E(-s)ds \dots (6)$$

Where $E(s)$ can be expressed as follows:

$$E(S) = \frac{N_{n-1}s^{n-1} + \dots + N_1 s + N_0}{D_n s^n + D_{n-1}s^{n-1} + \dots + D_1 s + D_0} \qquad \dots (7)$$

assuming type 1 behavior.

J follows from complex variable theory. To clarify the effect of system order, the subscript for J will be the system order. For an nth –order system.

$$J_n = (-1)^{n-1}\frac{B_n}{2D_n H_n} \qquad \dots (8)$$

Where H_n and B_n are determinants. H_n is the determinant of the $n \times n$ Hurwitz matrix. The first two rows of the Hurwitz matrix are formed from the coefficients of D(s), while the remaining rows consist of right-shifted versions of the first two rows until the $n \times n$ matrix is formed. Thus we write

$$H_n = \begin{bmatrix} D_{n-1} & D_{n-3}\dots\dots\dots & \\ D_n & D_{n-2}\dots\dots\dots & \\ 0 & D_{n-1} & D_{n-3}\dots \\ 0 & D_{n-2}\dots\dots\dots\dots & \\ \dots\dots\dots\dots\dots\dots\dots\dots & & \end{bmatrix} \qquad \dots (9)$$

The determinant B_n is found by first calculating

$$N(s)N(-s) = b_{2n-2}s^{2n-2} + \dots + b_2 s^2 + b_0 \qquad \dots (10)$$

Then first row of the Hurwitz matrix is replaced by the coefficients of $N(s)N(-s)$ while the remaining rows are unchanged.

$$B_n = \begin{bmatrix} b_{2n-2} & \ldots\ldots & b_2 \ldots b_0 \\ D_n & D_{n-2} \ldots\ldots\ldots & \\ 0 & D_{n-1} & D_{n-3} \ldots \\ 0 & D_n & D_{n-2} \ldots \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \end{bmatrix} \quad \ldots (11)$$

5. PARTICLE SWARM OPTIMIZATION (PSO) [6,7,8] FOR THE COST FUNCTION J_n

The cost function $\mathbf{J_n}$, a function of k will be minimized i.e. the reciprocal of the same will be maximized using PSO.

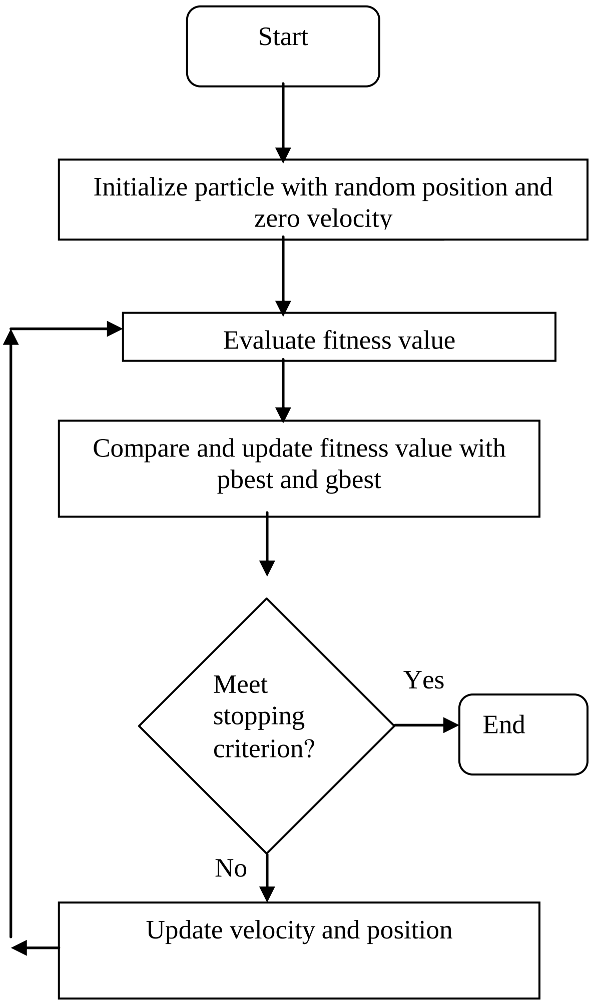

Fig .3: Flow chart for particle swarm optimization

pbest: the best solution (fitness) a particle has achieved so far, gbest: the global best solution for all particles. gbestk : the value of k for J_n is minimum.

6. SYSTEM DESIGN:

Under present situation, overall transfer function of the system is given by

$$\frac{Y(s)}{R(s)} = T(s)$$

$$= \frac{(ks+k_1)}{s^4+4s^3+4s^2+ks+k_1} .(12)$$

Thus for the entire system the characteristics equation is $s^4+4s^3+4s^2+ks+k_1=0$. For this characteristics equation to make the design problem with stability, the Routh array is constructed as below:

$$\begin{array}{llll} s^4 & 1 & 4 & k_1 \\ s^3 & 4 & k & \\ s^2 & 16-k & k_1 & \\ s^1 & \dfrac{16k-k^2-4k_1}{16-k} & 0 & \\ s^0 & k_1 & & \end{array}$$

For stability

$k_1 > 0$,let k_1=1, 16-k<0, i.e. k<16

Now

$$\frac{Y(s)}{R(s)} = T(s)$$

$$= \frac{(ks+1)}{s^4+4s^3+4s^2+ks+1} .(12)$$

$$T_E(s) = \frac{1-T(s)}{s}$$

$$= \frac{s^3+4s^2+4s}{s^4+4s^3+4s^2+ks+1} \quad \ldots(14)$$

$$N_3 = 1, N_2 = 4, \ N_1 = 4, \ N_0 = 0 \quad \ldots (15)$$

$$D_4 = 1, D_3 = 4 \ , D_2 = 4 \ , \quad \ldots(16)$$

$$D_1 = k \ , \quad D_0 = 1 \quad \ldots(17)$$

$$J_4 = -\frac{B_4}{2D_4H_4} \quad \ldots (18)$$

$$H_3 = \begin{bmatrix} D_3 & D_1 & 0 & 0 \\ D_4 & D_2 & D_0 & 0 \\ 0 & D_3 & D_1 & 0 \\ D_4 & D_2 & D_0 & 0 \end{bmatrix} \quad \ldots (19)$$

$$N(s) = s^3 + 4s^2 + 4s \quad \ldots (20)$$

$$N(-s) = -s^3 + 4s^2 - 4s \quad \ldots (21)$$

$$J_4 = -\frac{B_4}{2D_4H_4}$$

$$J_4 = \frac{(12k+60)}{(32k-32-2k^2)}$$

J_4 will be minimum for k=6 and the minimum value of the same is obtained by Particle swarm optimization method.

7. METHODS AND MATERIAL

So long any control system is considered in continuous data control system (continuous time domain ↔ Laplace domain), the system analysis and study get restricted for any change in the system parameter, or input variation for easy and ready study. To circumvent this problem sample data (s.d.) control system makes study and analysis easy and ready available with variation in system parameter and also the input. For this reason the system is also studied in sample data control model. The stability of the present system is tested by Jury's stability test which guarantees the stability of the overall system. Needless to mention, any stable system when operated in s.d. mode, the system is not necessarily to be guaranteed to remain stable in the s.d. mode also, there being the enhancement of the order of the system.

As any control system deserves to reach its steady state by which the system finally runs, and follows the input at that state, the designed parameter K is accordingly decided, the other desirable characteristic performances being also available in the system.

8. Matlab output files [8]

```
H4 =
[ 4, k, 0, 0]
[ 1, 4, 1, 0]
[ 0, 4, k, 0]
[ 0, 1, 4, 1]

 H =16*k-16-k^2

 B4 =
[  -1,   8, -16,   0]
[   1,   4,   1,   0]
[   0,   4,   k,   0]
[   0,   1,   4,   1]

B =-12*k-60
J4 =-(-12*k-60)/(32*k-32-2*k^2)
Pbestx=6
n =    12    60
d =    -2    32   -32
nd =  24        240       -2304
dd =   4       -128        1152       -2048        1024
k =   -16
        6
s =  -3.3117
     -0.2514 + 1.2506i
     -0.2514 - 1.2506i
     -0.1856
a =    -4    -4    -6    -1
        1     0     0     0
        0     1     0     0
        0     0     1     0
b =     1
        0
        0
        0
c =     0     0     6     1
d =     0
t = 0     0     6   -23
    0     6     1   -24
    6     1     0   -36
    1     0     0    -6
dobs =   4
t1 = 1     -4    12  -38
     0      1    -4   12
     0      0     1   -4
     0      0     0    1
rcont =     4

Transfer function:
         6 s + 1
-----------------------------
s^4 + 4 s^3 + 4 s^2 + 6 s + 1
 Margins =   1.7430    1.8257    5.8176    1.7256
Transfer function:
         6 s + 1
-----------------------------
s^4 + 4 s^3 + 4 s^2 + 6 s + 1

Transfer function:
0.00663 z^3 + 0.01558 z^2 - 0.01675 z - 0.004375
-----------------------------------------------
 z^4 - 3.322 z^3 + 4.127 z^2 - 2.253 z + 0.4493

Sampling time: 0.2
Enter length of output vector = 5
Type in the numerator coefficients =
   [0.00663  0.01558 -0.01675  -0.004375]
Type in the denominator coefficients =
    [1 -3.322  4.127  -2.253   0.4493]

Coefficients of the power series expansion
   0.0066    0.0376    0.0808    0.1238    0.1596
d =    1.0000   -3.3220    4.1270   -2.2530    0.4493
t =0.9250 + 0.2335i
   0.9250 - 0.2335i
   0.9553
   0.5168
v = 0.0013
z = 11.1513
s =0.4493   -2.2530    4.1270   -3.3220    1.0000
   1.0000   -3.3220    4.1270   -2.2530    0.4493
  -0.7981    2.3097   -2.2727    0.7604         0
   0.7604   -2.2727    2.3097    2.3097         0
   0.0588   -0.1152    0.0576         0         0
a0 =    0.4493 a4 =  1 b0 =    0.7981 b3 =    0.7604
```

c0 = 0.0588c2 = 0.0576

9. Output plots

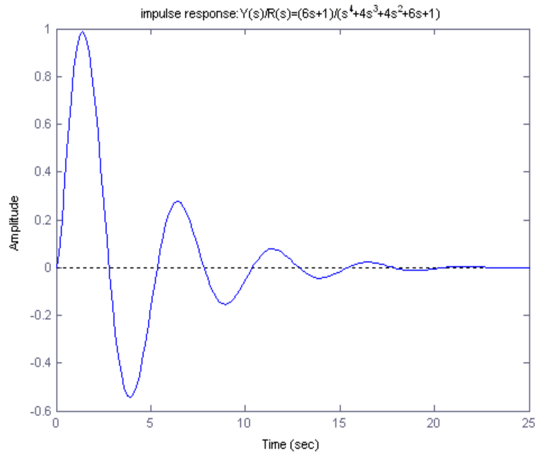

Figure 4

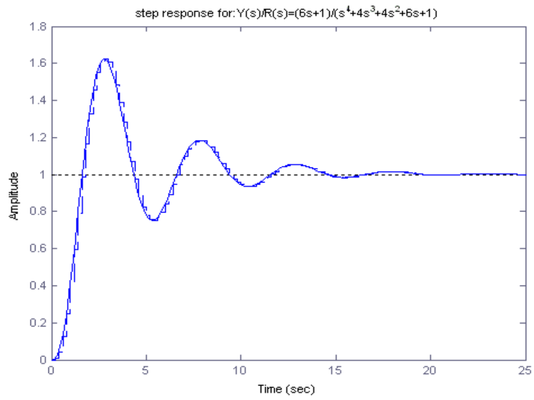

Figure 5

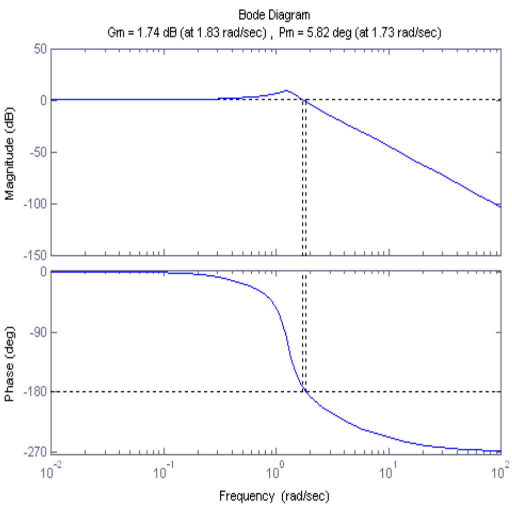

Figure 6

10. Conclusion

Since the system is stable in both continuous and sample data control system, controllable, observable and having appropriate gain and phase margin so the design of a blood pressure control system becomes feasible.

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